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BEM solutions to a class of elliptic BVPs for anisotropic trigonometrically graded media

St. N. Jabir¹, M. I. Azis^{2,*}, Z. Djafar³, B. Nurwahyu²

¹Department of Electrical Engineering, ATI Polytechnic, Makassar, Indonesia

²Department of Mathematics, Hasanuddin University, Makassar, Indonesia

³Department of Mechanical Engineering, Hasanuddin University, Makassar, Indonesia

E-mail: mohivanazis@yahoo.co.id (*Corresponding author)

Abstract. Boundary value problems (BVPs) governed by a class of elliptic equations for anisotropic trigonometrically graded media are solved using Boundary Element Method (BEM). The variable coefficient governing equation is transformed to a constant coefficient equation which is then transformed to a boundary integral equation. The results show the convergence, consistency, and accuracy of the BEM solutions.

1. Introduction

Several types of constant coefficient equations have been solved using BEM (see for examples [1, 2, 3, 4]). But in general this is not the case for variable coefficient equation. There is some progress in using BEM to solve several types of variable coefficient governing equations (see for examples [5, 6, 7, 8, 9, 10, 11, 12, 13])

The governing equation considered by Salam et. al in [11] takes the form

$$\frac{\partial}{\partial x_i} \left[\lambda_{ij}(x_1, x_2) \frac{\partial \zeta(x_1, x_2)}{\partial x_j} \right] = 0 \quad (1)$$

This paper is intended to extend the work by Salam et. al [11] for problems with governing equation (1) to for 2D boundary value problems governed by another type of (dimensionless) elliptic equation of the form

$$\frac{\partial}{\partial x_i} \left[\lambda_{ij}(x_1, x_2) \frac{\partial \zeta(x_1, x_2)}{\partial x_j} \right] + \beta(x_1, x_2) \zeta(x_1, x_2) = 0 \quad (2)$$

where the coefficients λ_{ij} depend on x_1 and x_2 and ¹² the repeated summation convention (summing from 1 to 2) is employed.

The matrix of coefficients $[\lambda_{ij}]$ is a real symmetric positive definite matrix so that equation (2) is a second order elliptic partial differential equation and may be written explicitly as

$$\frac{\partial}{\partial x_1} \left(\lambda_{11} \frac{\partial \zeta}{\partial x_1} \right) + 2 \frac{\partial}{\partial x_1} \left(\lambda_{12} \frac{\partial \zeta}{\partial x_2} \right) + \frac{\partial}{\partial x_2} \left(\lambda_{22} \frac{\partial \zeta}{\partial x_2} \right) + \beta \zeta = 0$$

Further, the coefficients λ_{ij} and β are required to be twice differentiable functions of the two independent variables x_1 and x_2 . The analysis here is specially relevant to an anisotropic medium

¹ but it equally applies to isotropic media. For isotropy, the coefficients in (2) take the form $\lambda_{11} = \lambda_{22}$ and $\lambda_{12} = 0$ and use of these equations in the following analysis immediately yields the corresponding results for an isotropic medium.

Steady infiltration problems (when $\beta < 0$, see for examples [14, 15]), acoustic problems (when $\beta > 0$, see for examples [16, 17]), and antiplane strain in elastostatics and plane thermostatic problems (when $\beta = 0$) are the areas for which the governing equation is of the type (2).

The technique of transforming (2) to a constant coefficient equation will again be used for obtaining a boundary integral equation for the solution of (2).

2. The boundary value problem ²

Ref²erred to a Cartesian frame Ox_1x_2 a solution to (2) is sought which is valid in a region Ω in R^2 with boundary $\partial\Omega$ which consists of a finite number of piecewise smooth closed curves. On $\partial\Omega_1$ the dependent variable $\varsigma(\mathbf{x})$ ($\mathbf{x} = (x_1, x_2)$) is specified and on $\partial\Omega_2$

$$P(\mathbf{x}) = \lambda_{ij} (\partial\varsigma/\partial x_j) n_i \quad (3)$$

is specified where $\partial\Omega = \partial\Omega_1 \cup \partial\Omega_2$ and $\mathbf{n} = (n_1, n_2)$ denotes the outward pointing normal to $\partial\Omega$.

3. The boundary integral equation ¹⁶

The boundary integral equation is derived by transforming the variable coefficient equation (2) to a constant coefficient equation. The coefficients λ_{ij} and β are required to take the form

$$\lambda_{ij}(\mathbf{x}) = \bar{\lambda}_{ij} g(\mathbf{x}) \quad (4)$$

$$\beta(\mathbf{x}) = \bar{\beta} g(\mathbf{x}) \quad (5)$$

where the $\bar{\lambda}_{ij}$ and $\bar{\beta}$ are constants and g is a differentiable function of \mathbf{x} . Use of (4) and (5) and in (2) yields

$$\bar{\lambda}_{ij} \frac{\partial}{\partial x_i} \left(g \frac{\partial \varsigma}{\partial x_j} \right) + \bar{\beta} g \varsigma = 0 \quad (6)$$

Let

$$\varsigma(\mathbf{x}) = g^{-1/2}(\mathbf{x}) \psi(\mathbf{x}) \quad (7)$$

so that (6) may be written in the form

$$\bar{\lambda}_{ij} \frac{\partial}{\partial x_i} \left[g \frac{\partial (g^{-1/2} \psi)}{\partial x_j} \right] + \bar{\beta} g^{1/2} \psi = 0$$

That is

$$\bar{\lambda}_{ij} \left[\left(\frac{1}{4} g^{-3/2} \frac{\partial g}{\partial x_i} \frac{\partial g}{\partial x_j} - \frac{1}{2} g^{-1/2} \frac{\partial^2 g}{\partial x_i \partial x_j} \right) \psi + g^{1/2} \frac{\partial^2 \psi}{\partial x_i \partial x_j} \right] + \bar{\beta} g^{1/2} \psi = 0 \quad (8)$$

Use of the identity

$$\frac{\partial^2 g^{1/2}}{\partial x_i \partial x_j} = -\frac{1}{4} g^{-3/2} \frac{\partial g}{\partial x_i} \frac{\partial g}{\partial x_j} + \frac{1}{2} g^{-1/2} \frac{\partial^2 g}{\partial x_i \partial x_j}$$

permits (8) to be written in the form

$$g^{1/2} \bar{\lambda}_{ij} \frac{\partial^2 \psi}{\partial x_i \partial x_j} - \psi \bar{\lambda}_{ij} \frac{\partial^2 g^{1/2}}{\partial x_i \partial x_j} + \bar{\beta} g^{1/2} \psi = 0 \quad (9)$$

If we further restrict the function $g(\mathbf{x})$ to take the exponential form

$$g(\mathbf{x}) = \{A [\cos(\alpha_m x_m) + \sin(\alpha_m x_m)]\}^2 \quad \bar{\beta} = -\bar{\lambda}_{ij} \alpha_i \alpha_j < 0 \quad (10)$$

where α_m are constant, then

$$\bar{\lambda}_{ij} \frac{\partial^2 g^{1/2}}{\partial x_i \partial x_j} - \bar{\beta} g^{1/2} = 0 \quad (11)$$

Substitution (11) into (9) implies a constant coefficients equation

$$\bar{\lambda}_{ij} \frac{\partial^2 \psi}{\partial x_i \partial x_j} = 0 \quad (12)$$

Also, substitution of (4) and (7) into (3) gives

$$P = -P_g \psi + P_\psi g^{1/2} \quad (13)$$

where

$$P_g(\mathbf{x}) = \bar{\lambda}_{ij} \frac{\partial g^{1/2}}{\partial x_j} n_i \quad P_\psi(\mathbf{x}) = \bar{\lambda}_{ij} \frac{\partial \psi}{\partial x_j} n_i$$

¹⁶ A boundary integral equation for the solution of (12) is given in the form

$$\kappa(\mathbf{x}_0) \psi(\mathbf{x}_0) = \int_{\partial\Omega} [\Gamma(\mathbf{x}, \mathbf{x}_0) \psi(\mathbf{x}) - \Phi(\mathbf{x}, \mathbf{x}_0) P_\psi(\mathbf{x})] ds(\mathbf{x}) \quad (14)$$

where $\mathbf{x}_0 = (a, b)$, $\kappa = 0$ if $(a, b) \notin \Omega \cup \partial\Omega$, $\kappa = 1$ if $(a, b) \in \Omega$, $\kappa = \frac{1}{2}$ if $(a, b) \in \partial\Omega$ and $\partial\Omega$ has a continuously turning tangent at (a, b) .

The so called fundamental solution Φ in (14) is any solution of the equation

$$\bar{\lambda}_{ij} \frac{\partial^2 \Phi}{\partial x_i \partial x_j} = \delta(\mathbf{x} - \mathbf{x}_0)$$

and the Γ is given by

$$\Gamma(\mathbf{x}, \mathbf{x}_0) = \bar{\lambda}_{ij} \frac{\partial \Phi(\mathbf{x}, \mathbf{x}_0)}{\partial x_j} n_i$$

where δ is the Dirac delta function. Following Azis in [18], for two-dimensional problems Φ and Γ are given by

$$\begin{aligned} \Phi(\mathbf{x}, \mathbf{x}_0) &= \frac{K}{2\pi} \ln R \\ \Gamma(\mathbf{x}, \mathbf{x}_0) &= \frac{K}{2\pi R} \bar{\lambda}_{ij} \frac{\partial R}{\partial x_j} n_i \end{aligned} \quad (15)$$

where

$$\begin{aligned} K &= \dot{\tau} / \zeta \\ \zeta &= [\bar{\lambda}_{11} + \bar{\lambda}_{12} \dot{\tau} + \bar{\lambda}_{22} (\dot{\tau}^2 + \dot{\tau}^2)] / 2 \\ R &= \sqrt{(\dot{x}_1 - \dot{a})^2 + (\dot{x}_2 - \dot{b})^2} \\ \dot{x}_1 &= x_1 + \dot{\tau} x_2 \\ \dot{a} &= a + \dot{\tau} b \\ \dot{x}_2 &= \dot{\tau} x_2 \\ \dot{b} &= \dot{\tau} b \end{aligned}$$

where $\bar{\tau}$ and $\bar{\tau}$ are respectively the real and the positive imaginary parts of the complex root τ of the quadratic

$$\bar{\lambda}_{11} + 2\bar{\lambda}_{12}\tau + \bar{\lambda}_{22}\tau^2 = 0$$

The derivatives $\partial R/\partial x_j$ needed for the calculation of the Γ in (15) are given by

$$\begin{aligned} \frac{\partial R}{\partial x_1} &= \frac{1}{R} (\dot{x}_1 - \dot{a}) \\ \frac{\partial R}{\partial x_2} &= \bar{\tau} \left[\frac{1}{R} (\dot{x}_1 - \dot{a}) \right] + \tau \left[\frac{1}{R} (\dot{x}_2 - \dot{b}) \right] \end{aligned}$$

Use of (7) and (13) in (14) yields

$$\begin{aligned} \kappa(\mathbf{x}_0) g^{1/2}(\mathbf{x}_0) \zeta(\mathbf{x}_0) &= \int_{\partial\Omega} \left\{ \left[g^{1/2}(\mathbf{x}) \Gamma(\mathbf{x}, \mathbf{x}_0) - P_g(\mathbf{x}) \Phi(\mathbf{x}, \mathbf{x}_0) \right] \zeta(\mathbf{x}) \right. \\ &\quad \left. - \left[g^{-1/2}(\mathbf{x}) \Phi(\mathbf{x}, \mathbf{x}_0) \right] P(\mathbf{x}) \right\} ds(\mathbf{x}) \end{aligned} \quad (16)$$

This equation provides a boundary integral equation for determining ζ and P at all points of Ω .

4. Numerical examples

In order to show the appropriateness of the BEM and the validity of the analysis used above for deriving the boundary integral equation (16), some particular boundary value problems will be solved. The integrals in equation (16) are evaluated numerically using the Bode's quadrature (see Abramowitz and Stegun [19]).

4.1. Examples with analytical solutions

In order to see the convergence and accuracy of the BEM we will consider some examples of problems with analytical solutions. The parameters for the trigonometrical inhomogeneity function $g(\mathbf{x})$ are $A = 3, \alpha_1 = 0.15, \alpha_2 = 0.55$. Plot of $g(\mathbf{x})$ is shown in Figure 1.

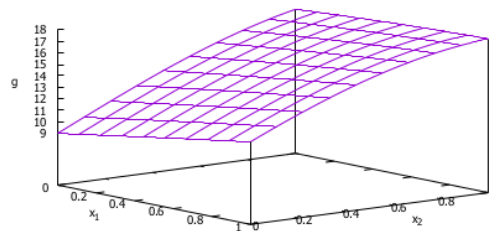


Figure 1. A trigonometrical inhomogeneity function $g(\mathbf{x}) = \{3 [\cos(0.15x_1 + 0.55x_2) + \sin(0.15x_1 + 0.55x_2)]\}^2$

The geometry of the region Ω and the boundary conditions are as depicted in Figure 2.

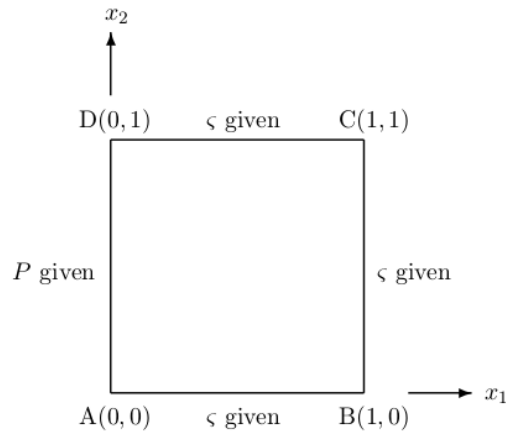


Figure 2. The geometry of all problems in Section 4.1

The values of the constant coefficients $\bar{\lambda}_{ij}$ for the governing equation (2) are

$$\bar{\lambda}_{11} = 0.25, \bar{\lambda}_{12} = 0.25, \bar{\lambda}_{22} = 1$$

Therefore from (9)

$$\bar{\beta} = -\bar{\lambda}_{ij}\alpha_i\alpha_j = -0.349375$$

The function $g(\mathbf{x})$ satisfies (9). Therefore equation (12) has to be the corresponding constant coefficient equation for $\psi(\mathbf{x})$. For test problems, we will take $\psi(\mathbf{x})$ as a linear function satisfying (12)

$$\psi(\mathbf{x}) = B(\gamma_0 + \gamma_1x_1 + \gamma_2x_2)$$

4.1.1. Problem 4.1.1 We choose analytical solution with parameters for $\psi(\mathbf{x})$

$$B = 2, \gamma_0 = 1, \gamma_1 = 0.85, \gamma_2 = 0.25$$

so that from (7)

$$\varsigma(\mathbf{x}) = 2(1 + 0.85x_1 + 0.25x_2) / \{3[\cos(0.15x_1 + 0.55x_2) + \sin(0.15x_1 + 0.55x_2)]\}$$

Table 1 shows the results of the analytical and BEM solutions with 20, 40 and 80 elements of equal length. The BEM solution converges to the analytical solution as the number of elements increases.

Table 1. BEM and analytical solutions for Problem 4.1.1

(x_1, x_2)	ζ	$\partial\zeta/\partial x_1$	$\partial\zeta/\partial x_2$	ζ	$\partial\zeta/\partial x_1$	$\partial\zeta/\partial x_2$
	BEM 20 elements			BEM 40 elements		
(.1,.5)	.6501	.3993	-.0567	.6489	.4012	-.0569
(.3,.5)	.7286	.3896	-.0636	.7282	.3921	-.0666
(.5,.5)	.8060	.3850	-.0707	.8059	.3854	-.0736
(.7,.5)	.8823	.3797	-.0753	.8824	.3795	-.0777
(.9,.5)	.9582	.3835	-.0795	.9580	.3773	-.0798
(.5,.1)	.8655	-.0026	-.2967	.8607	.3935	-.1999
(.5,.3)	.8262	.3973	-.1351	.8266	.3933	-.1353
(.5,.7)	.7970	.3861	-.0221	.7965	.3822	-.0211
(.5,.9)	.7999	.6441	.1031	.7976	.3931	.0378
	BEM 80 elements			Analytical		
(.1,.5)	.6486	.4020	-.0576	.6483	.4029	-.0587
(.3,.5)	.7280	.3931	-.0679	.7280	.3935	-.0692
(.5,.5)	.8059	.3856	-.0749	.8059	.3857	-.0762
(.7,.5)	.8824	.3794	-.0789	.8823	.3792	-.0800
(.9,.5)	.9578	.3756	-.0803	.9576	.3741	-.0809
(.5,.1)	.8616	.4044	-.2137	.8622	.4060	-.2159
(.5,.3)	.8269	.3934	-.1367	.8271	.3934	-.1382
(.5,.7)	.7963	.3825	-.0222	.7960	.3826	-.0233
(.5,.9)	.7967	.3823	.0262	.7962	.3840	.0248

4.1.2. Problem 4.1.2 We choose analytical solution with parameters

$$B = 4, \gamma_0 = 1, \gamma_1 = 0.85, \gamma_2 = 0.25$$

so that

$$\zeta(\mathbf{x}) = 4(1 + 0.85x_1 + 0.25x_2) / \{3[\cos(0.15x_1 + 0.55x_2) + \sin(0.15x_1 + 0.55x_2)]\}$$

Table 2 shows the results of the analytical and BEM solutions with 20, 40 and 80 elements of equal length. The BEM solution converges to the analytical solution as the number of elements increases.

Table 2. BEM and analytical solutions for Problem 4.1.2

(x_1, x_2)	ζ	$\partial\zeta/\partial x_1$	$\partial\zeta/\partial x_2$	ζ	$\partial\zeta/\partial x_1$	$\partial\zeta/\partial x_2$
	BEM 20 elements			BEM 40 elements		
(.1,.5)	1.3003	.7986	-.1134	1.2978	.8024	-.1138
(.3,.5)	1.4572	.7793	-.1272	1.4563	.7843	-.1331
(.5,.5)	1.6119	.7701	-.1415	1.6118	.7708	-.1473
(.7,.5)	1.7646	.7595	-.1505	1.7648	.7591	-.1555
(.9,.5)	1.9164	.7671	-.1591	1.9159	.7546	-.1597
(.5,.1)	1.7309	-.0053	-.5933	1.7213	.7870	-.3998
(.5,.3)	1.6524	.7946	-.2702	1.6532	.7866	-.2705
(.5,.7)	1.5939	.7722	-.0442	1.5931	.7645	-.0423
(.5,.9)	1.5997	1.2882	.2062	1.5952	.7861	.0756
	BEM 80 elements			Analytical		
(.1,.5)	1.2971	.8040	-.1152	1.2967	.8058	-.1174
(.3,.5)	1.4561	.7861	-.1358	1.4559	.7871	-.1384
(.5,.5)	1.6118	.7712	-.1499	1.6117	.7714	-.1524
(.7,.5)	1.7647	.7589	-.1578	1.7647	.7585	-.1600
(.9,.5)	1.9156	.7511	-.1607	1.9153	.7482	-.1618
(.5,.1)	1.7232	.8087	-.4273	1.7243	.8120	-.4318
(.5,.3)	1.6537	.7868	-.2735	1.6542	.7868	-.2764
(.5,.7)	1.5926	.7650	-.0444	1.5920	.7652	-.0467
(.5,.9)	1.5934	.7646	.0525	1.5924	.7681	.0497

4.2. Examples without analytical solutions

In this section we will consider some examples of problems without simple analytical solutions. We setup some problems for a homogeneous isotropic material by taking $\bar{\lambda}_{11} = \bar{\lambda}_{22} = 1, \bar{\lambda}_{12} = 0$ and with symmetrical boundary conditions. The aim is to see the consistency of the BEM of whether it produces symmetrical solutions.

4.2.1. Problem 4.2.1 For this problem we take $g(\mathbf{x}) = \{A[\cos(\alpha_i x_i) + \sin(\alpha_i x_i)]\}^2 = 1$ with $A = 1, \alpha_1 = 0, \alpha_2 = 0$ so that $\bar{\beta} = \bar{\lambda}_{ij} \alpha_i \alpha_j = 0$. And the symmetrical boundary conditions are as shown in Figure 3.

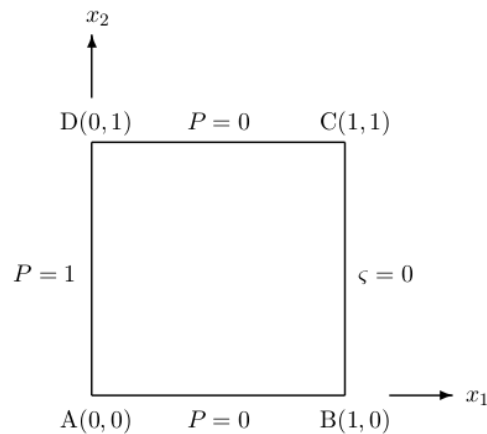


Figure 3. The geometry of Problem 4.2.1

Table 3 shows the results of the BEM solution using 40, 80, 160 and 320 elements of equal length. As expected, the results converge as the number of elements increases and also they are symmetrical about the axes $x_2 = 0.5$.

Table 3. BEM solution for Problem 4.2.1 ⁹

(x_1, x_2)	ζ	$\partial\zeta/\partial x_1$	$\partial\zeta/\partial x_2$	ζ	$\partial\zeta/\partial x_1$	$\partial\zeta/\partial x_2$
	BEM 40 elements			BEM 80 elements		
(.1,.5)	.8924	-.9973	.0000	.8967	-.9987	-.0000
(.3,.5)	.6932	-.9952	.0000	.6971	-.9980	-.0000
(.5,.5)	.4942	-.9943	.0000	.4975	-.9977	-.0000
(.7,.5)	.2955	-.9932	.0000	.2980	-.9973	-.0000
(.9,.5)	.0970	-.9919	.0000	.0986	-.9968	-.0000
(.5,.1)	.4940	-.9975	.0005	.4974	-.9982	.0002
(.5,.3)	.4942	-.9945	.0007	.4975	-.9978	.0002
(.5,.7)	.4942	-.9945	-.0007	.4975	-.9978	-.0002
(.5,.9)	.4940	-.9975	-.0005	.4974	-.9982	-.0002
	BEM 160 elements			BEM 320 elements		
(.1,.5)	.8985	-.9994	-.0000	.8993	-.9997	.0000
(.3,.5)	.6987	-.9991	-.0000	.6994	-.9996	.0000
(.5,.5)	.4989	-.9990	-.0000	.4995	-.9996	.0000
(.7,.5)	.2991	-.9989	-.0000	.2996	-.9995	.0000
(.9,.5)	.0993	-.9987	-.0000	.0997	-.9994	.0000
(.5,.1)	.4988	-.9992	.0001	.4995	-.9997	.0000
(.5,.3)	.4989	-.9991	.0001	.4995	-.9996	.0000
(.5,.7)	.4989	-.9991	-.0001	.4995	-.9996	-.0000
(.5,.9)	.4988	-.9992	-.0001	.4995	-.9997	-.0000

4.2.2. *Problem 4.2.2* Now we consider a problem with $g(\mathbf{x}) = 1$ again and the symmetrical boundary conditions are as shown in Figure 4. Table 4 shows the results of the BEM solution using 40, 80, 160 and 320 elements of equal length. The results converge as the number of elements increases and also they are symmetrical about the axes $x_1 = 0.5$.

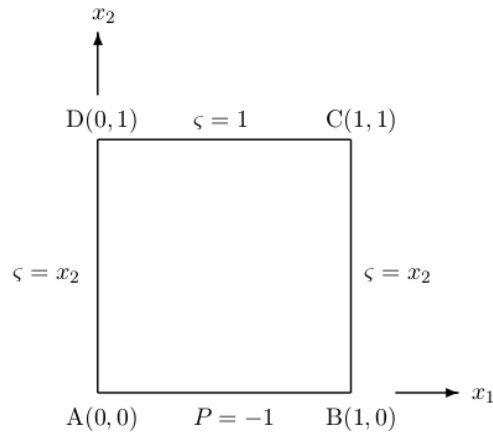


Figure 4. The geometry of Problem 4.2.2

Table 4. EM solution for Problem 4.2.2

(x_1, x_2)	ζ	$\partial\zeta/\partial x_1$	$\partial\zeta/\partial x_2$	ζ	$\partial\zeta/\partial x_1$	$\partial\zeta/\partial x_2$
	BEM 40 elements			BEM 80 elements		
(.1,.5)	.5003	.0031	1.0020	.5001	.0014	1.0001
(.3,.5)	.5008	.0016	1.0004	.5004	.0007	1.0003
(.5,.5)	.5010	-.0000	1.0005	.5004	.0000	1.0004
(.7,.5)	.5008	-.0016	1.0004	.5004	-.0007	1.0003
(.9,.5)	.5003	-.0031	1.0020	.5001	-.0014	1.0001
(.5,.1)	.1012	-.0000	.9987	.1005	-.0000	.9993
(.5,.3)	.3010	-.0000	.9992	.3004	-.0000	.9998
(.5,.7)	.7012	-.0000	1.0022	.7006	.0000	1.0012
(.5,.9)	.9019	.0000	1.0037	.9009	-.0000	1.0019
	BEM 160 elements			BEM 320 elements		
(.1,.5)	.5001	.0006	1.0001	.5000	.0003	1.0000
(.3,.5)	.5002	.0004	1.0002	.5001	.0002	1.0001
(.5,.5)	.5002	.0000	1.0002	.5001	-.0000	1.0001
(.7,.5)	.5002	-.0004	1.0002	.5001	-.0002	1.0001
(.9,.5)	.5001	-.0006	1.0001	.5000	-.0003	1.0000
(.5,.1)	.1002	.0000	.9996	.1001	-.0000	.9998
(.5,.3)	.3002	.0000	.9999	.3001	-.0000	.9999
(.5,.7)	.7003	.0000	1.0006	.7001	-.0000	1.0003
(.5,.9)	.9005	-.0000	1.0010	.9002	-.0000	1.0005

5. Conclusion

The scalar elliptic governing equation (2) is used for modeling physical problems such as steady infiltration problems (when $\beta < 0$), acoustic problems (when $\beta > 0$), and antiplane strain in elastostatics and plane thermostatic problems (when $\beta = 0$). The boundary integral equation (16) was derived from this governing equation (2) and straight from (16) a BEM was then constructed for calculation of numerical solutions to the problems for anisotropic trigonometrically graded media. The results show the convergence, consistency, and accuracy of the BEM solutions. Together with its ease in implementation, it may be concluded that BEM is a good numerical method for solving such kind of problems.

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